

Srednješolske enačbe

Relativna števila - Funkcije in grafi - Posebne funkcije -
Diferenciali - Integrali - Gibanje - Sile in gibanje - Deformacije -
Valovanje - Toplota - Molekule - Električna - E & M polje - Svetlobni
valovi

Relativna števila

Relativna števila $0, \pm 0,5, \pm 0,67, \pm 3,143 \dots$

$$|\pm p| = p$$

Računska pravila $(\pm p) + (+q) = (\pm p) + q$
 $(\pm p) + (-q) = (\pm p) - q$

$$(\pm p) - (+q) = (\pm p) - q$$
$$(\pm p) - (-q) = (\pm p) + q$$

$$(+p)(+q) = +pq$$
$$(+p)(-q) = -pq$$
$$(-p)(-q) = +pq$$

Skalarna potenca

$$p^{-n} = \frac{1}{p^n}$$
$$p^{m/n} = \sqrt[n]{p^m}$$
$$p^{-m/n} = \frac{1}{p^{m/n}}$$

Logaritem $b^L = N \Leftrightarrow L = \log_b N$

Računska pravila $\log pq = \log p + \log q$
 $\log \frac{p}{q} = \log p - \log q$
 $\log p^a = a \log p$

Desetiški logaritem $\lg p = \log_{10} p$

Funkcije in grafi

Sorazmernost $u = ax$

Obratna sorazmernost $u = \frac{a}{x}$

Potenčna funkcija $u = ax^n$

Linearna funkcija $u = ax + b$

Kvadratna funkcija $u = ax^2 + bx + c$
 $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Posebne funkcije

Geometrijska vrsta $G = 1 + x + x^2 + x^3 + \dots$
 $G = \frac{1}{1-x}, |x| < 1$

Binomski obrazec $B = (1+x)^n =$
 $1 + \frac{n}{1}x + \frac{n(n-1)}{1 \cdot 2}x^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3}x^3 + \dots + x^n$

Binomska vrsta $B = (1+x)^s =$
 $1 + \frac{s}{1}x + \frac{s(s-1)}{1 \cdot 2}x^2 + \frac{s(s-1)(s-2)}{1 \cdot 2 \cdot 3}x^3 + \dots; |x| < 1$

Potenčna vrsta $u = a_0 + a_1x + a_2x^2 + a_3x^3 \dots$
 $|u| < \infty \iff \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} x \right| < 1$

Eksponentna funkcija $\exp(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
 $e^x = \exp(x), \quad e = 2,72$

Logaritemska funkcija $u = e^x \iff \log_e u = x$

Naravni in desetiški logaritem $\ln u = \log_e u$
 $\lg u = \frac{\ln u}{\ln 10}, \quad \ln 10 = 2,30$
 $10^x = e^{x \ln 10}$

Kotne funkcije $\frac{y}{r} = \sin \varphi$
 $\frac{x}{r} = \cos \varphi$
 $\frac{y}{x} = \tan \varphi$

Polovični in dvojni kot $(\cos A)^2 = \frac{1}{2}(1 + \cos 2A)$
 $\sin A \cos A = \frac{1}{2} \sin 2A$

Vsota in razlika kotov

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B\end{aligned}$$

Sinusoida

$$\begin{aligned}u &= A \sin(kx + \delta) = a \sin kx + b \cos kx \\ a &= A \cos \delta \\ b &= A \sin \delta\end{aligned}$$

Obratne kotne funkcije

$$\begin{aligned}\sin x = u &\Leftrightarrow x = \arcsin u \\ \cos x = u &\Leftrightarrow x = \arccos u \\ \tan x = u &\Leftrightarrow x = \arctan u\end{aligned}$$

Diferenciali

Odvod

$$u' = \lim_{dx \rightarrow 0} \frac{u(x + dx) - u(x)}{dx}$$

Diferencial

$$\begin{aligned}du &= u' \cdot dx \\ \frac{du}{dx} &= u' \\ \frac{d}{dx} \left(\frac{du}{dx} \right) &= \frac{d^2u}{dx^2} = u''\end{aligned}$$

Odводи osnovnih funkcij

$$\begin{aligned}\frac{d}{dx} c &= 0 \\ \frac{d}{dx} x^n &= nx^{n-1} \\ \frac{d}{dx} x^s &= sx^{s-1} \\ \frac{d}{dx} e^x &= e^x \\ \frac{d}{dx} \sin x &= \cos x \\ \frac{d}{dx} \cos x &= -\sin x\end{aligned}$$

Odvod obratne funkcije

$$\begin{aligned}\frac{du}{dx} \cdot \frac{dx}{du} &= 1 \\ \frac{d}{dx} \ln x &= \frac{1}{x} \\ \frac{d}{dx} \arcsin x &= + \frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \arccos x &= - \frac{1}{\sqrt{1-x^2}}\end{aligned}$$

Odvod sestavljene funkcije

$$\frac{d}{dx}(cu) = c \frac{du}{dx}$$

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$\frac{d}{dx}(uv) = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{du/dx \cdot v - u \cdot dv/dx}{v^2}$$

$$\frac{d}{dx}u(v) = \frac{du}{dv} \cdot \frac{dv}{dx}$$

Razvoj v potenčno vrsto

$$u(x) = u(0) + \frac{u'(0)}{1!}x + \frac{u''(0)}{2!}x^2 + \dots$$

$$u(a+h) = u(a) + \frac{u'(a)}{1!}h + \frac{u''(a)}{2!}h^2 + \dots$$

Razvoj osnovnih funkcij

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots; |x| < 1$$

$$\ln \frac{1+x}{1-x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots\right); |x| < 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

Maksimum in minimum

$$u = \max \Leftrightarrow u' = 0 \text{ in } u'' < 0$$

$$u = \min \Leftrightarrow u' = 0 \text{ in } u'' > 0$$

Integrali

Integral

$$\Delta u = \lim_{du \rightarrow 0} \sum du = \int du$$

$$\int_a^b du = \int_a^b u' dx = u(b) - u(a)$$

$$\int u' dx = u(x) + C$$

Integrali osnovnih funkcij

$$\int c dx = cx$$

$$\int x^s dx = \frac{x^{s+1}}{s+1}; s \neq -1$$

$$\int \frac{1}{x} dx = \ln x$$

$$\int e^x dx = e^x$$

$$\int \sin x dx = -\cos x$$

$$\int \cos x \, dx = \sin x$$

Pravila integriranja

$$\int_a^b u \, dx = - \int_b^a u \, dx$$

$$\int_a^b u \, dx + \int_b^c u \, dx = \int_a^c u \, dx$$

$$\int cu \, dx = c \int u \, dx$$

$$\int (u \pm v) \, dx = \int u \, dx \pm \int v \, dx$$

$$\int u'v \, dx = uv - \int uv' \, dx$$

$$\int u(x) \, dx = \int u[v(x)] \, dv$$

Ploščina pod krivuljo

$$S = \int z \, dx$$

Prostornina rotacijskega telesa

$$V = \pi \int z^2 \, dx$$

Gibanje

Premo enakomerno gibanje

$$v = \frac{s}{t}$$

$$s = vt$$

Premo neenakomerno gibanje

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$s = \int v \, dt$$

$$v = \int a \, dt$$

Prosti pad

$$v = gt, \quad g = 9,8 \, \text{m/s}^2$$

$$s = \frac{gt^2}{2}$$

$$v^2 = 2gs$$

Gibanje v ravnini

$$v_x = \frac{dx}{dt}$$

$$v_z = \frac{dz}{dt}$$

$$v^2 = v_x^2 + v_z^2$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$a_z = \frac{dv_z}{dt} = \frac{d^2z}{dt^2}$$

$$a^2 = a_x^2 + a_z^2$$

Poševni met

$$v_x = v_{x0}$$

$$v_z = v_{z0} - gt$$

$$x = v_{x0}t$$

$$z = v_{z0}t - \frac{gt^2}{2}$$

Nihanje obešene kroglice

$$t_0 = 2\pi\sqrt{\frac{l}{g}}$$

$$x = x_0 \cos \omega t, \quad \omega = \frac{2\pi}{t_0} = 2\pi\nu$$

Kroženje obešene kroglice

$$x = r \cos \omega t$$

$$y = r \sin \omega t$$

$$v = \omega r$$

$$a_r = \omega^2 r = \frac{v^2}{r}$$

Orbitiranje planetov

$$\left(\frac{t}{t_0}\right)^2 = \left(\frac{r}{r_0}\right)^3$$

Sile in gibanje

Težni zakon

$$F_g = mg$$

Gostota

$$\rho = \frac{m}{V}$$

$$\sigma = \rho g$$

Gibalni zakon

$$F = ma = m \frac{dv}{dt}$$

$$F_x = m \frac{dv_x}{dt}$$

$$F_z = m \frac{dv_z}{dt}$$

Vrteči se sistem

$$F = m\omega^2 r$$

$$F = 2m\omega v$$

Izrek o kinetični energiji

$$\int F_{\parallel} ds = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = \Delta K$$

Izrek o mehanski energiji

$$\int F_{\text{other}\parallel} ds = \Delta K + \Delta W$$

Težna energija

$$W = mgh$$

Splošni težni zakon

$$F_g = \kappa \frac{mM}{r^2}, \quad \kappa \sim 10^{-11} \text{ Nm}^2/\text{kg}^2$$

Težno polje okrog delca

$$g = \kappa \frac{M}{r^2}$$

Težna energija dveh delcev

$$W = -\kappa \frac{mM}{r}$$

Orbitalni zakon

$$\frac{t^2}{r^3} = \frac{4\pi^2}{\kappa M}$$

Deformacije

Vzmet

$$F = ks$$

$$W = \frac{1}{2} ks^2$$

Spiralna vzmet

$$M = D\varphi$$

$$W = \frac{1}{2} D\varphi^2$$

Nateg trdnine

$$\frac{F_{\perp}}{S} = E \frac{\Delta l}{l}$$

Strig trdnine

$$\frac{F_{\parallel}}{S} = G\alpha$$

Stiskanje tekočine

$$\Delta p = K \frac{\Delta V}{V}$$

Izotermni plinski zakon

$$pV = p_0V_0$$

Strig tekočine

$$\frac{F_{\parallel}}{S} = \eta \frac{\Delta v}{l}$$

Turbulentni količnik

$$R = \frac{l\nu\rho}{\eta}$$

Masni in volumski tok

$$\Phi_m = \frac{dm}{dt}$$

$$\Phi_V = \frac{dV}{dt}$$

$$\Phi_m = \rho\Phi_V$$

Viskozni tok po cevi

$$\Phi_V = \frac{\pi r^4 \Delta p}{8 \eta l}$$

Viskozni upor kroglice

$$F = 6\pi\eta rv$$

Enačba tokovnice

$$p + \rho gh + \frac{\rho v^2}{2} = \text{const}$$

Dinamični upor telesa

$$F = \frac{1}{2} c \rho S v^2$$

Površinska napetost

$$\frac{F}{l} = \gamma$$

Tlak v kapljici

$$\Delta p = \frac{2\gamma}{r}$$

Valovanje

Val na vrvi

$$s = f(x - ct)$$

$$c^2 = \frac{F}{\mu}$$

Potujoč harmonični val

$$s = s_0 \cos k(x - ct) = s_0 \cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi\nu$$

$$c = \lambda\nu = \frac{\omega}{k}$$

Stoječ harmonični val

$$s = 2s_0 \cos kx \cos \omega t$$

Valovi v globoki vodi

$$c^2 = \frac{g\lambda}{2\pi} = \frac{g}{k}$$

Valovi v plitvi vodi

$$c^2 = gH$$

Odbojni zakon	$\alpha_r = \alpha_i$
Lomni zakon	$\frac{\sin \alpha_i}{\sin \alpha_t} = \frac{c_i}{c_t}$
Uklon na dveh režah	$d \sin \alpha = N\lambda, \quad N = 0, 1, 2 \dots$
Zvočni valovi	$c^2 = \frac{K}{\rho}$
Bližanje oddajnika	$\frac{\nu'}{\nu} = 1 + \frac{v}{c}$
Bližanje k oddajniku	$\frac{\nu'}{\nu} = \frac{1}{1 - v/c}$
Valovno čelo	$\sin \theta = \frac{c}{v}$
Gostota energije	$w = \frac{W}{V}$
Za harmonično komponento	$w = \frac{1}{2} \rho v_0^2 = \frac{1}{2} \rho \omega^2 s_0^2$
Energijski tok	$P = \frac{W}{t}$
Gostota toka	$j = \frac{P}{S}$
	$j = cw$
Točkast izvor	$j = \frac{P/\Omega}{r^2} = \frac{I}{r^2}$

Toplota

Temperatura	T
Izohorni plinski zakon	$\frac{T}{T_0} = \frac{p}{p_0}$
Temperaturno raztezanje teles	$\frac{dl}{l} = \alpha dT$

	$\frac{dV}{V} = \beta dT$
	$\beta = 3\alpha$
Plinska enačba stanja	$pV = mRT$
Zakon o meh. segrevanju	$cm\Delta T = F_g h$
Notranja energija snovi	$\Delta U = cm\Delta T$
Toplota in energijski zakon	$\Delta U = A + Q$
Toplotni stik snovi	$c_1 m_1 \Delta T_1 = c_2 m_2 \Delta T_2$
Raztežno delo plina	$A_{\text{exp}} = -\int p dV$
	$\Delta U = A_{\text{other}} + A_{\text{exp}} + Q$
Segrevanje plina pri stalni prostornini	$\Delta U = Q = mc_V \Delta T$
Segrevanje plina pri stalnem tlaku	$\Delta U = Q - p\Delta V = mc_p \Delta T$
	$c_p - c_V = R$
Adiabatno stiskanje in raztezanje plina	$TV^{\kappa-1} = \text{const}$
	$\kappa = \frac{c_p}{c_V}$
Taljenje	$Q = q_t m$
Izparevanje	$Q = q_i m$
Sežiganje	$Q = q_s m$
Nasičeni parni tlak nad tekočino	$p_s = p_0 \exp \frac{q_i}{R} \left(\frac{1}{T_0} - \frac{1}{T} \right)$
Vlažni zrak	$f = \frac{\rho_v}{\rho_{vs}} = \frac{p_v}{p_{vs}}$
	$r = \frac{\rho_v}{\rho} = \frac{R}{R_v} \frac{p_v}{p}$
Toplotni tok	$P = \frac{Q}{t}$

Gostota toka $j = \frac{P}{S}$

Zakon o prevajanju toplote $j = \lambda \frac{\Delta T}{l}$

Molekule

Molekulska masa m_1

Atomska masna enota $m_1 = \mu u$

Kilomolsko število $N_A \cdot u = 1 \text{ kg}, N_A \sim 10^{27}, u \sim 10^{-27} \text{ kg}$

Kilomolska masa snovi $M = N_A \mu u = \mu \cdot 1 \text{ kg}$

Splošna plinska konstanta $R = \frac{R^*}{M}, R^* = 8300 \text{ J/K}$

$$pV = nR^*T, n = \frac{m}{M}$$

Elektrika

Električni naboj e

Električni tok $I = \frac{e}{t}$

Gostota toka $j = \frac{I}{S}$

Elektrolizni zakon $\frac{m}{e} = \frac{1}{F} \frac{M}{Z}, F = 96 \cdot 10^6 \text{ C}$

Osnovni naboj $F = N_A e_0, e_0 \sim 10^{-19} \text{ C}$

Zakon o el. segravanju $mc_V \Delta T \propto It \propto A$

Delo in gonilna napetost $A = U_g e$

Zaporedna vezava $U_g = U_{g1} + U_{g2}$

Padci napetosti $U_g = \sum U_k$

Električna moč $P = UI$

Zakon o el. uporu	$U = RI$
Specifični upor	$R = \xi \frac{l}{S}$
Specifična prevodnost	$\sigma = \frac{1}{\xi}$
Zaporedna vezava	$R = R_1 + R_2$
Vzporedna vezava	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$
Trošena moč v uporniku	$P = RI^2 = U^2/R$
Notranji upor člena	$U_g = (R_N + R)I$

E & M polje

El. sila in el. polje	$F_e = eE$
Polje v kondenzatorju	$E = \frac{U}{l}$ $\frac{e}{S} = \varepsilon_0 \frac{U}{l}, \quad \varepsilon_0 = 8,85 \cdot 10^{-12} \text{ As/Vm}$
Kapaciteta kondenzatorja	$e = CU$ $C = \varepsilon_0 \frac{S}{l}$
Snov v kondenzatorju	$C_{\text{filled}} = \varepsilon C$
Mag. sila in mag. polje	$F_m = BI$
Mag. navor na zanko	$M_m = BIS$
Kinematična indukcija napetosti	$U_i = B_{\perp} \frac{dS}{dt}$
Dinamična indukcija napetosti	$U_i = S \frac{dB_{\perp}}{dt}$
Indukcijski zakon	$U_i = \frac{d}{dt} (B_{\perp} S)$

Polje v tuljavi	$B = \mu_0 \frac{NI}{l}, \quad \mu_0 = 1,26 \cdot 10^{-6} \text{ Vs/Am}$
Induktivnost tuljave	$U_i = L \frac{dI}{dt}$ $L = \mu_0 N^2 \frac{S}{l}$
Generator izmenične napetosti	$U = U_0 \sin \omega t$ $U_0 = NBS\omega$
Transformator	$\frac{U_1}{U_2} = \frac{N_1}{N_2}$ $U_1 I_1 = U_2 I_2$
Efektivna napetost in tok	$\langle P \rangle = RI_{\text{ef}}^2 = \frac{U_{\text{ef}}^2}{R}$ $I_{\text{ef}} = \frac{I_0}{\sqrt{2}}$ $U_{\text{ef}} = \frac{U_0}{\sqrt{2}}$
Upor upornika	$U_{\text{ef}} = RI_{\text{ef}}$
Upor kondenzatorja	$U_{\text{ef}} = \frac{1}{C\omega} I_{\text{ef}}$
Upor tuljave	$U_{\text{ef}} = L\omega I_{\text{ef}}$
Energija el. polja v kondenzatorju	$W = \frac{1}{2} CU^2$
Energija mag. polja v tuljavi	$W = \frac{1}{2} LI^2$
Gostota energije	$w = \frac{W}{V}$
Gostota energije el. polja	$w = \frac{1}{2} \epsilon_0 E^2$
Gostota energije mag. polja	$w = \frac{1}{2\mu_0} B^2$

Nihajni krog $\omega^2 = \frac{1}{LC}$

Svetlobni valovi

Svetlobni tok $P = \frac{Q}{t}$

Gostota svetlobnega toka $j = \frac{P}{S_{\perp}}$

Osvetljenost ploskve $j' = j \cos \alpha$

Točkast izotropni izvor $j = \frac{P}{4\pi r^2}$

Njegova svetilnost $I = \frac{dP}{d\Omega}$
 $j = \frac{I}{r^2}$

Svetlost razsežnega izvora $B = \frac{dI}{dS_{\perp}}$

Njegova gostota izseva $j^* = \frac{dP}{dS}$

Sevanje črnega telesa $j^* = \sigma T^4, \quad \sigma = 5,67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$

Njegov maksimum $\lambda_{\max} = \frac{b}{T}, \quad b = 2,9 \cdot 10^{-3} \text{ Km}$

Njegov spekter $\frac{dj^*}{d\lambda} = \frac{c_1}{\lambda^5} \frac{1}{\exp(c_2/\lambda T) - 1}$

$$c_1 = 3,74 \cdot 10^{-16} \text{ Wm}^2$$

$$c_2 = 1,44 \cdot 10^{-2} \text{ Km}$$

Odbojnost $j^* = aj'$

Absorpcija $j = j_0 \exp(-\mu l)$

Relativna magnituda zvezd $m_1 - m_2 = -2,5 \lg \frac{j_1}{j_2}$

$$m = -2,5 \lg \frac{j}{j_0}, \quad j_0 = 2,5 \cdot 10^{-8} \text{ W/m}^2$$