Visokošolske enačbe

Kompleksna števila – Vektorji in matrike – Večkratne funkcije – Krivulje in ploskve – Prostorska polja – Statistika – Dinamika – Relativnost – Termokinetika – Statična E & M polja – Elektromagnetni valovi – Elektroni in ioni – Kvantni delci – Valovna mehanika – Molekule, kristali, plini – Atomska jedra – Zvezde in vesolje

Kompleksna števila

Fazorji

$$\hat{u} = (u_1, u_2) = (u\cos\varphi, u\sin\varphi)$$
$$u = \sqrt{({u_1}^2 + {u_2}^2)} = |\hat{u}|$$
$$\varphi = \operatorname{atan}\frac{u_2}{u_1} = \operatorname{Arg}(\hat{u})$$
$$\operatorname{Re}(\hat{u}) = u_1$$
$$\operatorname{Im}(\hat{u}) = u_2$$

Računska pravila

$$c\hat{u} = (cu_1, cu_2)$$
$$\hat{u} + \hat{v} = (u_1 + v_1, u_2 + v_2)$$
$$\hat{u}\,\hat{v} = uv\,(\cos(\alpha+\beta), \sin(\alpha+\beta))$$
$$\hat{u}\,/\,\hat{v} = (u/v)\,(\cos(\alpha-\beta), \sin(\alpha-\beta))$$
$$\hat{u}\hat{v} = (u_1v_1 - u_2v_2, u_1v_2 + u_2v_1)$$
$$\hat{u}\,/\,\hat{v} = (u_1v_1 + u_2v_2, u_2v_1 - u_1v_2)$$

Imaginarna enota

$$\hat{u} = u_1\cdot(1,0) + u_2\cdot(0,1)$$
$$\hat{u} = u_1 + \mathrm{i}u_2$$
$$\mathrm{i} = (0,1)$$
$$\mathrm{i}\cdot\mathrm{i} = \mathrm{i}^2 = -1$$

Konjugirani fazorji

$$\hat{u}^* = u_1 - \mathrm{i}u_2$$
$$|\hat{u}|^2 = \hat{u}\hat{u}^*$$

Potenca in eksponencial

$$\hat{u}^n = \hat{u}\cdot\hat{u}\ldots\hat{u}$$
$$\hat{u}^n = u^n(\cos n\varphi + \mathrm{i}\sin n\varphi)$$
$$\hat{u}^s = u^s(\cos s\varphi + \mathrm{i}\sin s\varphi)$$
$$\mathrm{e}^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\sin\varphi$$
$$\mathrm{e}^{\hat{u}} = \mathrm{e}^{u_1+\mathrm{i}u_2} = \mathrm{e}^{u_1}\mathrm{e}^{\mathrm{i}u_2} = \mathrm{e}^{u_1}(\cos u_2 + \mathrm{i}\sin u_2)$$

Kompleksne kotne funkcije

$$\cos\varphi = \frac{\mathrm{e}^{\mathrm{i}\varphi} + \mathrm{e}^{-\mathrm{i}\varphi}}{2}$$
$$\sin\varphi = \frac{\mathrm{e}^{\mathrm{i}\varphi} - \mathrm{e}^{-\mathrm{i}\varphi}}{2\mathrm{i}}$$
$$\cos\hat{u} = \frac{\mathrm{e}^{\mathrm{i}\hat{u}} + \mathrm{e}^{-\mathrm{i}\hat{u}}}{2}$$
$$\sin\hat{u} = \frac{\mathrm{e}^{\mathrm{i}\hat{u}} - \mathrm{e}^{-\mathrm{i}\hat{u}}}{2\mathrm{i}}$$

Kompleksne funkcije skalarja

$$\hat{u}(t) = u_1(t) + \mathrm{i}u_2(t)$$

Diferencial in integral

$$\frac{\mathrm{d}\hat{u}}{\mathrm{d}t} = \frac{\mathrm{d}u_1}{\mathrm{d}t} + \mathrm{i}\,\frac{\mathrm{d}u_2}{\mathrm{d}t}$$

$$\int \hat{u}\,\mathrm{d}t = \int u_1\,\mathrm{d}t + \mathrm{i}\int u_2\,\mathrm{d}t$$

Harmonične vrste

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,\mathrm{d}t$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,\mathrm{d}t, \quad n = 1, 2, 3\ldots$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,\mathrm{d}t, \quad n = 1, 2, 3\ldots$$

$$\frac{1}{T}\int_0^T f(t)^2\,\mathrm{d}t = {a_0}^2 + \frac{1}{2}\sum_{n=1}^{\infty} ({a_n}^2 + {b_n}^2)$$

Kompleksne harmonične vrste

$$f(t) = \mathrm{Re}\sum_{n=-\infty}^{\infty} \hat{A}_n\,\mathrm{e}^{\mathrm{i}n\omega t}$$

$$\hat{A}_n = \frac{1}{T}\int_0^T f(t)\,\mathrm{e}^{-\mathrm{i}n\omega t}\,\mathrm{d}t, \quad n = 0, \pm 1, \pm 2\ldots$$

$$\frac{1}{T}\int_0^T f(t)^2\,\mathrm{d}t = \sum_{n=-\infty}^{\infty} |\hat{A}_n|^2$$

Harmonični integrali

$$f(t) = \mathrm{Re}\int_{-\infty}^{\infty} \hat{A}(\omega)\,\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}\omega$$

$$\hat{A}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\,\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)^2\,\mathrm{d}t = \int_{-\infty}^{\infty} |\hat{A}(\omega)|^2\,\mathrm{d}\omega$$

Vektorji in matrike

Vektor premika

$$\boldsymbol{r} = (x, y, z)$$

$$r^2 = x^2 + y^2 + z^2$$

Zasuk koordinatnega sistema okoli osi z

$$x' = +x\cos\varphi + y\sin\varphi$$
$$y' = -x\sin\varphi + y\cos\varphi$$
$$z' = z$$

$$x'^2 + y'^2 + z'^2 = x^2 + y^2 + z^2$$

Razteg in vsota

$$\lambda\boldsymbol{u} = (\lambda u_1, \lambda u_2, \lambda u_3)$$
$$\boldsymbol{u} + \boldsymbol{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$$

Enotni vektorji

$$\boldsymbol{e}_1 = (1, 0, 0)$$
$$\boldsymbol{e}_2 = (0, 1, 0)$$
$$\boldsymbol{e}_3 = (0, 0, 1)$$
$$\boldsymbol{u} = u_1\boldsymbol{e}_1 + u_2\boldsymbol{e}_2 + u_3\boldsymbol{e}_3 = \sum u_i\boldsymbol{e}_i$$

Skalarni produkt

$$\boldsymbol{u}\cdot\boldsymbol{v} = uv\cos\varphi$$
$$\boldsymbol{u}\cdot\boldsymbol{v} = u_1v_1 + u_2v_2 + u_3v_3$$
$$\boldsymbol{u}\cdot\boldsymbol{u} = {u_1}^2 + {u_2}^2 + {u_3}^2 = u^2$$

Vektorski produkt

$$\boldsymbol{u}\times\boldsymbol{v} = uv\sin\varphi\cdot\boldsymbol{n}$$
$$\boldsymbol{u}\times\boldsymbol{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$$

Sorazmernost vektorjev

$$u_1 = A_{11}x_1 + A_{12}x_2 + A_{13}x_3$$
$$u_2 = A_{21}x_1 + A_{22}x_2 + A_{23}x_3$$
$$u_3 = A_{31}x_1 + A_{32}x_2 + A_{33}x_3$$

$$u_i = \sum_j A_{ij}x_j$$

Sorazmernostna matrika

$$\begin{vmatrix} u_1 \\ u_2 \\ u_3 \end{vmatrix} = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \cdot \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$$

$$\boldsymbol{u} = \boldsymbol{A}\cdot\boldsymbol{x}$$

Posebne matrike

$$\boldsymbol{I} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\boldsymbol{D} = \begin{vmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{vmatrix}$$

$$\boldsymbol{R}_3 = \begin{vmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Računske operacije

$$\lambda\boldsymbol{A} = \boldsymbol{B} \Longleftrightarrow B_{ij} = \lambda A_{ij}$$
$$\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{C} \Longleftrightarrow C_{ij} = A_{ij} + B_{ij}$$
$$\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{C} \Longleftrightarrow C_{ij} = \sum_k A_{ik} B_{kj}$$

Inverzna matrika

$$\boldsymbol{A} \cdot \boldsymbol{x} = \boldsymbol{u} \Longleftrightarrow \boldsymbol{x} = \boldsymbol{A}^{-1} \cdot \boldsymbol{u}$$
$$\boldsymbol{A}^{-1} \cdot \boldsymbol{A} = \boldsymbol{I}$$

$$[I_{ij}]^{-1} = \operatorname{diag}(1, 1, 1)$$
$$[D_{ij}]^{-1} = \operatorname{diag}(1/\lambda_1, 1/\lambda_2, 1/\lambda_3)$$
$$[R_{ij}]^{-1} = [R_{ji}]$$

Nje izračun

$$[\boldsymbol{A} \mid \boldsymbol{I}] \rightarrow [\boldsymbol{I} \mid \boldsymbol{A}^{-1}]$$

Večkratne funkcije

Vektorska funkcija skalarja

$$\boldsymbol{u}(t) = [u_1(t), u_2(t), u_3(t)]$$

Diferencial in integral

$$\boldsymbol{u}' = \lim_{\mathrm{d}t \to 0} \frac{\boldsymbol{u}(t + \mathrm{d}t) - \boldsymbol{u}(t)}{\mathrm{d}t}$$
$$\boldsymbol{u}' = (u_1', u_2', u_3')$$
$$\mathrm{d}\boldsymbol{u} = \boldsymbol{u}' \cdot \mathrm{d}t$$
$$\mathrm{d}\boldsymbol{u} = (\mathrm{d}u_1, \mathrm{d}u_2, \mathrm{d}u_3)$$

$$\boldsymbol{u}(t) = \boldsymbol{u}(0) + \frac{\boldsymbol{u}'(0)}{1!}\, t + \frac{\boldsymbol{u}''(0)}{2!}\, t^2 + \ldots$$

$$\boldsymbol{u}(t_0 + h) = \boldsymbol{u}(t_0) + \frac{\boldsymbol{u}'(t_0)}{1!}\, h + \frac{\boldsymbol{u}''(t_0)}{2!}\, h^2 + \ldots$$

$$\boldsymbol{u} = \int \boldsymbol{u}' \,\mathrm{d}t = (\int u_1' \,\mathrm{d}t, \int u_2' \,\mathrm{d}t\,, \int u_3' \,\mathrm{d}t)$$

Skalarna funkcija več spremenljivk

$$u = u(x, y)$$

Parcialni odvodi

$$u_x = \lim_{\mathrm{d}x \to 0} \frac{u(x + \mathrm{d}x, y) - u(x, y)}{\mathrm{d}x}$$

Totalni diferencial

$$\mathrm{d}u = (\mathrm{d}u)_x + (\mathrm{d}u)_y = u_x \,\mathrm{d}x + u_y \,\mathrm{d}y$$
$$\frac{(\mathrm{d}u)_x}{\mathrm{d}x} = \frac{\partial u}{\partial x} = u_x$$

Verižna odvajanja

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\partial u}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial u}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}$$
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

Implicitno odvajanje

$$F(x,y,u) = 0 \Longrightarrow F_x + F_u \frac{\partial u}{\partial x} = 0$$

Razvoj v potenčno vrsto

$$u(x, y) = u(0, 0) + xu_x + yu_y + \frac{1}{2}(x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}) + \ldots$$

$$u(a+h,b+k) = u(a,b) + \frac{1}{1!}(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y})u + \frac{1}{2!}(h\frac{\partial}{\partial x} + k\frac{\partial}{\partial y})^2u + \ldots$$

Ekstremi

$$u = \max \Longleftrightarrow \mathrm{u}_{xx} < 0 \text{ in } u_{xx}u_{yy} - {u_{xy}}^2 > 0$$
$$u = \min \Longleftrightarrow \mathrm{u}_{xx} > 0 \text{ in } u_{xx}u_{yy} - {u_{xy}}^2 > 0$$

Vezani ekstremi

$$\varphi(x, y) = 0$$
$$u_x + \lambda\varphi_x = 0$$
$$u_y + \lambda\varphi_y = 0$$

Ploščinski integrali

$$U = \int u \,\mathrm{d}S$$
$$U = \int\int u \,\mathrm{d}x \,\mathrm{d}y = \int_c^d \mathrm{d}y \int_a^b u \,\mathrm{d}x = \int_a^b \mathrm{d}x \int_c^d u \,\mathrm{d}y$$
$$U = \int\int u\rho \,\mathrm{d}\rho \,\mathrm{d}\varphi$$

Prostorninski integrali

$$U = \int u \,\mathrm{d}V.$$

$$U = \int\int\int u \,\mathrm{d}x \,\mathrm{d}y \,\mathrm{d}z$$
$$U = \int\int\int u\,\rho \,\mathrm{d}\rho \,\mathrm{d}\varphi \,\mathrm{d}z$$
$$U = \int\int\int u\, r^2 \sin\theta \,\mathrm{d}r \,\mathrm{d}\varphi \,\mathrm{d}\theta$$

Krivulje in ploskve

Razdalja med točkama

$$s^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$$

Premica

$$y = kx$$

$$x = At$$
$$y = Bt$$

Krožnica

$$x^2 + y^2 = r^2$$

$$x = r\cos t$$
$$y = r\sin t$$

Elipsa

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$x = a\cos t$$
$$y = b\sin t$$

Hiperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$x = a/\cos t$$
$$y = b\tan t$$

Parabola

$$2py = x^2$$

$$x = At$$
$$y = Bt^2$$

Vektorski opis krivulje

$$\boldsymbol{r}(t) = (x(t), y(t))$$
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y'}{x'}$$
$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{x'y'' - y'x''}{x'^3}$$

Dolžina krivulje

$$\mathrm{d}s = |\mathrm{d}\boldsymbol{r}| = |\boldsymbol{r}'|\,\mathrm{d}t = \sqrt{(x'^2 + y'^2)}\,\mathrm{d}t$$
$$s = \int \sqrt{(x'^2 + y'^2)}\,\mathrm{d}t$$

Tangenta

$$\boldsymbol{\tau} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s}$$
$$\boldsymbol{\tau} = \frac{(x', y')}{\sqrt{(x'^2 + y'^2)}}$$

Ukrivljenost

$$K = |\frac{\mathrm{d}\boldsymbol{\tau}}{\mathrm{d}s}|$$
$$K = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{3/2}}$$
$$K = \frac{1}{R}$$

Ravnina

$$z = k_1x + k_2y$$

Krogla

$$x^2 + y^2 = r^2$$

$$x = R\sin\theta\cos\varphi$$
$$y = R\sin\theta\sin\varphi$$
$$z = R\cos\theta$$

Vektorski opis ploskve

$$\boldsymbol{r}(u,v) = (x(u,v), y(u,v), z(u,v))$$

Dolžina krivulje na ploskvi

$$\mathrm{d}s^2 = \boldsymbol{r}_u{}^2\mathrm{d}u^2 + 2\boldsymbol{r}_u\boldsymbol{r}_v\mathrm{d}u\mathrm{d}v + \boldsymbol{r}_v{}^2\mathrm{d}v^2 =$$
$$g_{11}\mathrm{d}u^2 + 2g_{12}\mathrm{d}u\mathrm{d}v + g_{22}\mathrm{d}v^2$$

$$g_{11} = x_u{}^2 + y_u{}^2 + z_u{}^2$$
$$g_{12} = x_u x_v + y_u y_v + z_u z_v$$
$$g_{22} = x_v{}^2 + y_v{}^2 + z_v{}^2$$

$$s = \int \sqrt{(g_{11}u'^2 + 2g_{12}u'v' + g_{22}v'^2)}\,\mathrm{d}t$$

Ploščina ploskve

$$\mathrm{d}S = \mathrm{d}_u s\,\mathrm{d}_v s\sin\alpha = \sqrt{(g_{11}g_{22} - g_{12}{}^2)}\mathrm{d}u\mathrm{d}v$$
$$S = \int \sqrt{(g_{11}g_{22} - g_{12}{}^2)}\,\mathrm{d}u\mathrm{d}v$$
$$S = \int \sqrt{(1 + z_x{}^2 + z_y{}^2)}\,\mathrm{d}x\mathrm{d}y$$

Normala

$$\boldsymbol{n} = \frac{\boldsymbol{r}_u \times \boldsymbol{r}_v}{|\boldsymbol{r}_u \times \boldsymbol{r}_v|} = \frac{\boldsymbol{r}_u \times \boldsymbol{r}_v}{\sqrt{(g_{11}g_{22} - g_{12}{}^2)}}$$
$$\boldsymbol{n} = \frac{(-z_x, -z_y, 1)}{\sqrt{(1 + z_x{}^2 + z_y{}^2)}}$$

Lok na krogli

$$\cos\alpha = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\cos(\varphi_2 - \varphi_1)$$

Hipotenuzni izrek

$$\cos d = \cos a\cos h$$

Kosinusni izrek

$$\cos a = \cos b\cos c + \sin b\sin c\cos A$$

Sinusni izrek

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Prostorska polja

Skalarno in vektorsko polje

$$U = U(x,y,z)$$
$$\boldsymbol{v} = (v_x(x,y,z), v_y(x,y,z), v_z(x,y,z))$$

Gradient polja

$$\mathrm{grad}\,U = \boldsymbol{n}\cdot\frac{\mathrm{d}U}{\mathrm{d}s}$$
$$\mathrm{grad}\,U = \left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right) = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)U = \boldsymbol{\nabla}U$$
$$\mathrm{d}U = \boldsymbol{\nabla}U\cdot\mathrm{d}\boldsymbol{r} = (\mathrm{d}\boldsymbol{r}\cdot\boldsymbol{\nabla})U$$
$$U_2 - U_1 = \int \boldsymbol{\nabla}U\,\mathrm{d}\boldsymbol{s}$$

Pretok in divergenca

$$\Phi = \int \boldsymbol{v}\cdot\boldsymbol{n}\,\mathrm{d}S$$

$$\int \boldsymbol{v}\cdot\boldsymbol{n}\,\mathrm{d}S = \int v_x\,\mathrm{d}y\,\mathrm{d}z + \int v_y\,\mathrm{d}z\,\mathrm{d}x + \int v_z\,\mathrm{d}x\,\mathrm{d}y$$

$$\operatorname{div}\boldsymbol{v} = \lim_{V\to 0}\frac{1}{V}\oint \boldsymbol{v}\cdot\boldsymbol{n}\,\mathrm{d}S$$

$$\operatorname{div}\boldsymbol{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = \boldsymbol{\nabla}\cdot\boldsymbol{v}$$

$$\oint \boldsymbol{v}\cdot\boldsymbol{n}\,\mathrm{d}S = \int \boldsymbol{\nabla}\cdot\boldsymbol{v}\,\mathrm{d}V$$

Cirkulacija in rotor

$$\Gamma = \oint \boldsymbol{v}\,\mathrm{d}\boldsymbol{s}$$

$$\oint \boldsymbol{v}\,\mathrm{d}\boldsymbol{s} = \oint v_x\,\mathrm{d}x + \oint v_y\,\mathrm{d}y + \oint v_z\,\mathrm{d}z$$

$$\operatorname{rot}\boldsymbol{v} = \boldsymbol{n}\cdot\lim_{S\to 0}\frac{1}{S}\oint \boldsymbol{v}\,\mathrm{d}\boldsymbol{s}$$

$$\operatorname{rot}\boldsymbol{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}, \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}, \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) = \boldsymbol{\nabla}\times\boldsymbol{v}$$

$$\oint \boldsymbol{v}\,\mathrm{d}\boldsymbol{s} = \int (\boldsymbol{\nabla}\times\boldsymbol{v})\cdot\boldsymbol{n}\,\mathrm{d}S$$

Operacije drugega reda

$$\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}U) = \boldsymbol{\nabla}^2 U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}U) = 0$$

$$\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\boldsymbol{v}) = 0$$

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{v}) = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{v}) - \boldsymbol{\nabla}^2\boldsymbol{v}$$

Statistika

Število permutacij

$$P_n = n!$$

Število variacij

$$V_n{}^r = \frac{n!}{(n-r)!}$$

Število kombinacij

$$C_n{}^r = \frac{n!}{r!\,(n-r)!}$$

Verjetnost izidov

$$P_k = \lim_{N\to\infty}\frac{N_k}{N}$$

$$\sum P_k = 1$$

Gostota verjetnosti

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \lim_{N\to\infty}\frac{\mathrm{d}N(x \pm \mathrm{d}x/2)}{N} = p(x)$$

$$\int \mathrm{d}P = \int p(x)\,\mathrm{d}x = 1$$

Verjetnost izida A ali B

$$P(\mathrm{A}\cup\mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B})$$

Verjetnost izida A in B

$$P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A}) \cdot P(\mathrm{B})$$

Binomska porazdelitev

$$P(n) = \frac{N!}{n!(N-n)!}\, p^n (1-p)^{N-n} = \mathrm{B}_{N,p}(n)$$

Normalna porazdelitev

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \frac{1}{\sigma\sqrt{2\pi}} \cdot \mathrm{e}^{-(x-\mu)^2/2\sigma^2} = \mathrm{G}_{\mu,\sigma}(x)$$

Prehod na novo spremenljivko

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \frac{\mathrm{d}P}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}z}$$

Povprečje

$$\langle x \rangle = \int x\, p(x)\, \mathrm{d}x$$

Varianca

$$\sigma_x{}^2 = \int (x - \langle x \rangle)^2\, p(x)\, \mathrm{d}x$$
$$\sigma_x{}^2 = \int x^2\, p(x)\, \mathrm{d}x - (\int x\, p(x)\, \mathrm{d}x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

Porazdelitev po več spremenljivkah

$$\frac{\mathrm{d}^2 P}{\mathrm{d}x\,\mathrm{d}y} = \lim_{N\to\infty} \frac{\mathrm{d}N(x \pm \mathrm{d}x/2, y \pm \mathrm{d}y/2)}{N} = p(x,y)$$

Kumulativna verjetnost

$$\frac{\mathrm{d}P}{\mathrm{d}x} = \int p(x,y)\, \mathrm{d}y = u(x)$$

Neodvisni spremenljivki

$$p(x,y) = u(x)\, v(y)$$

Povprečje in varianca

$$\langle x \rangle = \int x\, p(x,y)\, \mathrm{d}x\, \mathrm{d}y$$
$$\sigma_x{}^2 = \int (x - \langle x \rangle)^2\, p(x,y)\, \mathrm{d}x\, \mathrm{d}y$$

Kovarianca

$$\sigma_{xy} = \int (x - \langle x \rangle)(y - \langle y \rangle)\, p(x,y)\, \mathrm{d}x\, \mathrm{d}y$$

Korelacijski koeficient

$$r = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

Povprečje povprečij

$$\langle \bar{x} \rangle = \langle x \rangle$$

Varianca povprečij

$$\sigma_{\bar{x}}{}^2 = \frac{\sigma_x{}^2}{N}$$

Ocena natančnosti meritev

$$\langle x \rangle \approx \bar{x} \pm \frac{s_x}{\sqrt{N}}$$

Širjenje napak

$$u = cx \Longrightarrow \mathrm{d}u = |c|\, \mathrm{d}x$$
$$u = x \pm y \Longrightarrow \mathrm{d}u = \mathrm{d}x + \mathrm{d}y$$
$$u = xy \Longrightarrow \frac{\mathrm{d}u}{|u|} = \frac{\mathrm{d}x}{|x|} + \frac{\mathrm{d}y}{|y|}$$

$$u = \frac{x}{y} \Longrightarrow \frac{\mathrm{d}u}{|u|} = \frac{\mathrm{d}x}{|x|} + \frac{\mathrm{d}y}{|y|}$$
$$u = x^n \Longrightarrow \frac{\mathrm{d}u}{|u|} = |n| \, \frac{\mathrm{d}x}{|x|}$$
$$u = u(x) \Longrightarrow \mathrm{d}u = |u'| \, \mathrm{d}x$$
$$u = u(x, y) \Longrightarrow \mathrm{d}u^2 = (u_x \, \mathrm{d}x)^2 + (u_y \, \mathrm{d}y)^2$$

Dinamika

Lega
$$\boldsymbol{r} = (x, y, z)$$

Premik
$$\mathrm{d}\boldsymbol{r} = \mathrm{d}\boldsymbol{s} = (\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$$

Hitrost
$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t} = (v_x, v_y, v_z)$$

Pospešek
$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = (a_x, a_y, a_z)$$

Kotni zasuk
$$\mathrm{d}\boldsymbol{\varphi} = \boldsymbol{e}_r \times \frac{\mathrm{d}\boldsymbol{s}}{r}$$

Kotna hitrost
$$\boldsymbol{\omega} = \frac{\mathrm{d}\boldsymbol{\varphi}}{\mathrm{d}t}$$

Kotni pospešek
$$\boldsymbol{\alpha} = \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t}$$

Dolžinske in kotne povezave
$$\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$$
$$\boldsymbol{a}_\mathrm{t} = \boldsymbol{\alpha} \times \boldsymbol{r}$$
$$\boldsymbol{a}_\mathrm{r} = -\omega^2 \boldsymbol{r}$$
$$\boldsymbol{a}^2 = \boldsymbol{a}_\mathrm{t}{}^2 + \boldsymbol{a}_\mathrm{r}{}^2$$

Gibalni zakon za delec
$$\boldsymbol{F} = m \, \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = m \, \frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}$$

Izrek o kinetični energiji
$$\int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{s} = \frac{m\boldsymbol{v}_2{}^2}{2} - \frac{m\boldsymbol{v}_1{}^2}{2}$$

Gibalni zakon za sistem delcev
$$\boldsymbol{F} = m \, \frac{\mathrm{d}^2\boldsymbol{r}^*}{\mathrm{d}t^2}$$

Lega težišča
$$\boldsymbol{r}^* = \frac{1}{m} \sum m_i \, \boldsymbol{r}_i$$

Hitrost težišča

$$\boldsymbol{v}^* = \frac{\mathrm{d}\boldsymbol{r}^*}{\mathrm{d}t}$$

Gibalna količina sistema

$$\boldsymbol{G} = \sum m_i \boldsymbol{v}_i = m\boldsymbol{v}^*$$

Izrek o gibalni količini

$$\int \boldsymbol{F}\,\mathrm{d}t = \Delta \boldsymbol{G}$$

Sila curka

$$F = \Phi_m v$$

Raketna enačba

$$v - v_0 = u \ln \frac{m_0}{m}$$

Vrtilna količina

$$\boldsymbol{L} = \sum \boldsymbol{r}_i \times m_i \boldsymbol{v}_i$$

Izrek o vrtilni količini

$$\int \boldsymbol{M}\,\mathrm{d}t = \Delta \boldsymbol{L}$$

$$\boldsymbol{L} = \boldsymbol{r}^* \times m\boldsymbol{v}^* + \boldsymbol{L}^*$$

Kinetična energija

$$K = \sum \frac{1}{2} m_i \boldsymbol{v}_i^2$$

Izrek o kinetični energiji

$$A_{\mathrm{ext}} + A_{\mathrm{int}} = \Delta K$$

$$K = \frac{1}{2} m\boldsymbol{v}^{*2} + K^*$$

Vrtenje togega sistema

$$L = J\omega$$

$$J = \sum m_i r_i^2$$

$$M = \frac{\mathrm{d}J\omega}{\mathrm{d}t} = J\frac{\mathrm{d}\omega}{\mathrm{d}t}$$

$$K = \frac{1}{2} J\omega^2$$

$$J = J^* + mr^{*2}$$
$$L = L^* + mr^{*2}\omega$$
$$K = K^* + \frac{1}{2} mr^{*2}\omega^2$$

Težno nihanje telesa

$${\omega_0}^2 = \frac{mgl}{ml^2 + J^*}$$

Sučno nihanje telesa

$$\omega_0{}^2 = \frac{D}{J^*}$$

Prosto nihanje

$$u'' + \omega_0{}^2\, u = 0$$

$$u = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$$

Vzbujeno nihanje

$$u'' + \omega_0{}^2\, u = A \cos(\omega t + \delta)$$

$$u = u_0 \cos(\omega t + \delta)$$
$$u_0 = \frac{A}{\sqrt{(\omega_0{}^2 - \omega^2)}}$$

Vzbujeno nihanje z dušenjem

$$u'' + \gamma u' + \omega_0 u = A \cos(\omega t + \delta)$$

$$u = u_0 \cos(\omega t + \delta + \theta)$$

$$u_0 = \frac{A}{\sqrt{[(\omega^2 - \omega_0{}^2)^2 + \gamma^2\, \omega^2]}}$$

$$\tan\theta = \frac{-\gamma\omega}{\omega_0{}^2 - \omega^2}$$

Dušeno nihanje

$$u'' + \gamma u' + \omega_0{}^2 u = 0$$

$$u = u_0\, \mathrm{e}^{-\gamma t/2} \cos(\omega t + \delta)$$
$$\omega = \sqrt{(\omega_0{}^2 - \gamma^2)},\ \gamma < \omega_0$$

Gravitacijsko polje

$$\boldsymbol{F}_\mathrm{g} = m\boldsymbol{g}$$

Polje okrog masne točke

$$\boldsymbol{g}_\mathrm{P} = -\kappa \frac{m_\mathrm{Q}}{r_\mathrm{PQ}{}^2}\, \boldsymbol{n}_\mathrm{PQ}, \quad \kappa = 6{,}7 \cdot 10^{-11}\,\mathrm{Nm^2/kg^2}$$

Superpozicija polj

$$\boldsymbol{g}_\mathrm{P} = -\sum_\mathrm{Q} \kappa \frac{m_\mathrm{Q}}{r_\mathrm{PQ}{}^2}\, \boldsymbol{n}_\mathrm{PQ}$$

Gravitacijska energija

$$W - W_0 = -\int m\boldsymbol{g} \cdot \mathrm{d}\boldsymbol{s}$$

Gravitacijski potencial

$$\phi - \phi_0 = -\int \boldsymbol{g} \cdot \mathrm{d}\boldsymbol{s}$$

Potencial masne točke

$$\phi_\mathrm{P} = -\kappa \frac{m_\mathrm{Q}}{r_\mathrm{QP}}$$

Superpozicija potencialov

$$\phi_{\mathrm{P}} = -\sum_{\mathrm{Q}} \kappa \frac{m_{\mathrm{Q}}}{r_{\mathrm{QP}}}$$

Gibanje planeta

$$L = mr^2\varphi'$$

$$E = \frac{m}{2}(r'^2 + r^2\varphi'^2) - \frac{\alpha}{r}$$

$$r = \frac{p}{1 + \varepsilon\cos\varphi}$$
$$p = \frac{L^2}{m\alpha}$$
$$\varepsilon^2 = 1 + \frac{2EL^2}{m\alpha^2}$$

$$\frac{T^2}{a^3} = \frac{4\pi^2}{\kappa M}$$

Relativnost

Stalnost svetl. hitrosti

$$c' = c$$

Relativistični faktor

$$\gamma = \frac{1}{\sqrt{(1 - u^2/c^2)}}$$

Transformacija koordinat

$$t' = \gamma(t - ux/c^2)$$
$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$

Transformacija intervalov

$$\Delta t = t_{\mathrm{B}} - t_{\mathrm{A}}$$
$$\Delta x = x_{\mathrm{B}} - x_{\mathrm{A}}$$

$$\Delta t' = \gamma(\Delta t - u\Delta x/c^2)$$
$$\Delta x' = \gamma(\Delta x - u\Delta t)$$

Podaljšanje časa

$$\Delta t = \gamma\Delta t_0$$

Skrajšanje dolžin

$$\Delta l = \Delta l_0 / \gamma$$

Transformacija hitrosti

$$v = \frac{x_{\mathrm{B}} - x_{\mathrm{A}}}{t_{\mathrm{B}} - t_{\mathrm{A}}}$$

$$v_x' = \frac{v_x - u}{1 - v_x u/c^2}$$

$$v_y' = \frac{1}{\gamma}\ \frac{v_y}{1 - v_x u/c^2}$$
$$v_z' = \frac{1}{\gamma}\ \frac{v_z}{1 - v_x u/c^2}$$

Frekvenčni zamik svetlobe

$$\frac{\nu'}{\nu} = \frac{\sqrt{(1 - u/c)}}{\sqrt{(1 + u/c)}}$$

Sprememba valovne dolžine

$$\frac{\lambda'}{\lambda} = \frac{\sqrt{(1 + u/c)}}{\sqrt{(1 - u/c)}}$$

Približek za majhne hitrosti

$$\frac{\Delta\lambda}{\lambda} = \frac{u}{c}$$

Gibalna količina

$$\boldsymbol{G} = m\gamma(v)\boldsymbol{v}$$

Gibalni zakon

$$\frac{\mathrm{d}\boldsymbol{G}}{\mathrm{d}t} = \boldsymbol{F}$$

Polna energija

$$E = \gamma(v)\, mc^2$$

Kinetična in masna energija

$$E = K + mc^2$$
$$K = (\gamma(v) - 1)mc^2$$

Energijska invarianta

$$E^2 - (cG)^2 = (mc^2)^2$$

Transformacija G in E

$$G_x' = \gamma(G_x - uE/c^2)$$

$$E' = \gamma(E - uG_x)$$

$$\Delta G_x' = \gamma(\Delta \mathrm{G}_x - u\Delta E/c^2)$$
$$\Delta E' = \gamma(\Delta E - u\Delta G_x)$$

Gibalna količina svetlobe

$$G = \frac{E}{c}$$

Gostota gibalne količine

$$g = \frac{G}{V}$$

$$g = \frac{j}{c^2}$$

Svetlobni tlak

$$p = gc$$

$$p = (1 + R)\, gc$$

Termokinetika

Številska gostota molekul

$$n = \frac{N}{V}$$

Plinski zakon

$$p = nkT, \quad k = \frac{R^*}{N_\mathrm{A}}$$

Kinetični model tlaka

$$p = \frac{1}{3}\, n m_1 \langle v^2 \rangle$$

Kinetični model temperature

$$\langle \frac{1}{2}\, m_1 v^2 \rangle = \frac{3}{2}\, kT$$

Porazdelitev molekul po višini

$$\frac{dP}{\mathrm{d}z} = A \mathrm{e}^{-m_1 g z / kT}, \quad A = \frac{m_1 g}{kT}$$

Porazdelitev molekul po hitrosti

$$\frac{\mathrm{d}P}{\mathrm{d}v_x} = A\, \mathrm{e}^{-m_1 {v_\mathrm{x}}^2 / 2kT}$$

$$\frac{\mathrm{d}P}{\mathrm{d}v} = A^3 4\pi v^2 \mathrm{e}^{-m_1 v^2 / 2kT}, \quad A = (\frac{m_1}{2\pi kT})^{1/2}$$

Porazdelitev molekul po energiji

$$P_i = \frac{1}{Z}\, \mathrm{e}^{-E_i/kT}, \quad Z = \sum \mathrm{e}^{-E_i/kT}$$

$$\mathrm{d}P = \frac{1}{Z}\, \mathrm{e}^{-E/kT}\, \mathrm{d}E, \quad Z = \int \mathrm{e}^{-E/kT} \mathrm{d}E$$

Porazdelitev po faznem prostoru

$$\mathrm{d}P = \frac{1}{Z}\, \mathrm{e}^{-E(\boldsymbol{r},\boldsymbol{v})/kT}\, \mathrm{d}^3\boldsymbol{r}\mathrm{d}^3\boldsymbol{v}, \quad Z = \int \mathrm{e}^{-E(\boldsymbol{r},\boldsymbol{v})/kT}\, \mathrm{d}^3\boldsymbol{r}\mathrm{d}^3\boldsymbol{v}$$

Notranja energija plina

$$U = N \frac{f}{2}\, kT$$

Toplotna kapaciteta plina

$$C_V = \frac{U}{T} = N \frac{f}{2}\, k$$

Specifične toplote plina

$$c_V = \frac{C_V}{N m_1} = \frac{f}{2} \frac{k}{m_1} = \frac{f}{2} \frac{R^*}{M}$$

$$c_p = c_V + \frac{R^*}{M} = (\frac{f}{2} + 1)\, \frac{R^*}{M}$$

$$\kappa = \frac{c_p}{c_V} = 1 + \frac{2}{f}$$

Kristalna snov

$$U = 3NkT$$
$$c_V = 3\,\frac{R^*}{M}$$

Izhlapevanje tekočine

$$n_\mathrm{v} = \frac{1}{V_0}\,\mathrm{e}^{-W/kT}$$

Ionizacija plina

$$\frac{n_\mathrm{e} n_\mathrm{i}}{n_\mathrm{a}} = \frac{1}{V_0}\,\mathrm{e}^{-W/kT}$$

Prosta pot molekul

$$\langle l\rangle = \frac{1}{\pi(2r)^2 n}$$

Difuzivnost

$$j_m = -D\,\frac{\mathrm{d}\rho}{\mathrm{d}x}\,,\quad D = \frac{1}{3}\,\langle v\rangle\langle l\rangle$$

Toplotna prevodnost

$$j_Q = -\lambda\,\frac{\mathrm{d}T}{\mathrm{d}x}\,,\quad \lambda = \frac{1}{6}\,\langle v\rangle\langle l\rangle nfk$$

Viskoznost

$$\eta = \frac{1}{3}\,\langle v\rangle\langle l\rangle nm_1$$

Transportni koeficienti

$$\eta = \rho D = \lambda/c_V$$

Difuzijska enačba

$$\frac{\partial\rho}{\partial t} = D\,\frac{\partial^2\rho}{\partial x^2}$$

Difuzija tanke plasti

$$\rho(x,t) = \frac{1}{\sigma_x\sqrt{(2\pi)}}\,\exp\frac{-x^2}{2{\sigma_x}^2}\,,\quad {\sigma_x}^2 = 2Dt$$

Difuzijska enačba toplote

$$\frac{\partial T}{\partial t} = D^*\,\frac{\partial^2 T}{\partial x^2}\,,\quad D^* = \frac{\lambda}{c_p\rho}$$

Pijančeva hoja

$$\langle r^2\rangle = 6Dt$$

Prisilna hitrost

$$v_\mathrm{drift} = \frac{\langle\tau\rangle}{m_1}\,F = \mu F$$

Difuzivnost in gibljivost

$$D = \mu kT$$

Difuzija kroglic v tekočini

$$D = \frac{kT}{6\pi\eta r}$$

Statična E & M polja

Električna sila in polje

$$\boldsymbol{F}_\mathrm{e} = e\boldsymbol{E}$$

Polje točkastega naboja

$$\boldsymbol{E}_\mathrm{P} = \kappa_\mathrm{e} \frac{e_\mathrm{Q}}{{r_\mathrm{QP}}^2} \boldsymbol{n}_\mathrm{QP}$$

Superpozicija polj

$$\boldsymbol{E}_\mathrm{P} = \kappa_\mathrm{e} \sum_\mathrm{Q} \frac{e_\mathrm{Q}}{{r_\mathrm{QP}}^2} \boldsymbol{n}_\mathrm{QP}$$

$$\boldsymbol{E}_\mathrm{P} = \kappa_\mathrm{e} \int \frac{\rho_\mathrm{Q}\,\mathrm{d}V_\mathrm{Q}}{{r_\mathrm{QP}}^2} \boldsymbol{n}_\mathrm{QP}$$

Pretok in divergenca polja

$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{S} = 4\pi\kappa_\mathrm{e} e = \frac{e}{\varepsilon_0}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$

Cirkulacija in rotor polja

$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s} = 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = 0$$

Električni potencial

$$\boldsymbol{E} = -\boldsymbol{\nabla} U$$

$$U_\mathrm{B} - U_\mathrm{A} = -\int_\mathrm{A}^\mathrm{B} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{s}$$

Potencial točkastega naboja

$$U_\mathrm{P} = \frac{1}{4\pi\varepsilon_0} \frac{e_\mathrm{Q}}{r_\mathrm{QP}}$$

Superpozicija potencialov

$$U_\mathrm{P} = \frac{1}{4\pi\varepsilon_0} \sum_\mathrm{Q} \frac{e_\mathrm{Q}}{r_\mathrm{QP}}$$

$$U_\mathrm{P} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_\mathrm{Q}\,\mathrm{d}V_\mathrm{Q}}{r_\mathrm{QP}}$$

Potencialna enačba

$$\boldsymbol{\nabla}^2 U = -\frac{\rho}{\varepsilon_0}$$

Električni dipol

$$\boldsymbol{p}_\mathrm{e} = e\boldsymbol{d}$$

$$U = \frac{1}{4\pi\varepsilon_0} \frac{\boldsymbol{p}_\mathrm{e} \cdot \boldsymbol{e}_r}{r^2}$$

Navor na dipol

$$\boldsymbol{M} = \boldsymbol{p}_\mathrm{e} \times \boldsymbol{E}$$

Sila na dipol

$$\boldsymbol{F} = (\boldsymbol{p}_\mathrm{e} \cdot \boldsymbol{\nabla})\boldsymbol{E}$$

Energija dipola

$$W = -\boldsymbol{p}_\mathrm{e} \cdot \boldsymbol{E}$$

Polarizacija snovi

$$\boldsymbol{P} = \frac{\mathrm{d}\boldsymbol{p}_\mathrm{e}}{\mathrm{d}V} = ne\boldsymbol{d}$$

Električna susceptibilnost

$$\boldsymbol{P} = \chi_\mathrm{e}\, \varepsilon_0 \boldsymbol{E}$$

$$\varepsilon = 1 + \chi_\mathrm{e}$$

Magnetna sila in polje

$$\boldsymbol{F}_\mathrm{m} = I\boldsymbol{l} \times \boldsymbol{B}$$

Polje tokov

$$\boldsymbol{B}_\mathrm{P} = \kappa_\mathrm{m} \oint \frac{I\, \mathrm{d}\boldsymbol{s}_\mathrm{Q} \times \boldsymbol{n}_\mathrm{QP}}{r^2{}_\mathrm{QP}}$$

$$\boldsymbol{B}_\mathrm{P} = \kappa_\mathrm{m} \oint \frac{\boldsymbol{j}_\mathrm{Q} \times \boldsymbol{n}_\mathrm{QP}}{r_\mathrm{QP}{}^2}\, \mathrm{d}V_\mathrm{Q}$$

Pretok in divergenca polja

$$\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{S} = 0$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$

Cirkulacija in rotor polja

$$\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = 4\pi\kappa_\mathrm{m} I = \mu_0 I$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{j}$$

Magnetni potencial

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

Potencialna enačba

$$\boldsymbol{\nabla}^2 \boldsymbol{A} = -\mu_0 \boldsymbol{j}$$

Potencial tokov

$$\boldsymbol{A}_\mathrm{P} = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{j}_\mathrm{Q}\, \mathrm{d}V_\mathrm{Q}}{r_\mathrm{QP}}$$

Magnetni dipol

$$\boldsymbol{p}_\mathrm{m} = I\boldsymbol{S}$$

$$\boldsymbol{A} = \frac{\mu_0}{4\pi} \frac{\boldsymbol{p}_\mathrm{m} \times \boldsymbol{e}_r}{r^2}$$

Navor na dipol

$$\boldsymbol{M} = \boldsymbol{p}_\mathrm{m} \times \boldsymbol{B}$$

Sila na dipol

$$\boldsymbol{F} = \boldsymbol{\nabla}\,(\boldsymbol{p}_\mathrm{m} \cdot \boldsymbol{B}) = (\boldsymbol{p}_\mathrm{m} \cdot \boldsymbol{\nabla})\boldsymbol{B}$$

Energija dipola

$$W = -\boldsymbol{p}_\mathrm{m} \cdot \boldsymbol{B}$$

Magnetizacija snovi

$$\boldsymbol{M} = \frac{\mathrm{d}\boldsymbol{p}_\mathrm{m}}{\mathrm{d}V} = nI\boldsymbol{S}$$

Magnetna susceptibilnost

$$\boldsymbol{M} = \frac{\chi_\mathrm{m}}{\mu_0}\boldsymbol{B}$$

$$\frac{1}{\mu} = 1 - \chi_\mathrm{m}$$

Relativnost gostote nabojev in tokov

$$\rho' = \gamma(\rho - uj_x/c^2)$$
$$j'_x = \gamma(j_x - u\rho)$$
$$j'_y = j_y$$
$$j'_z = j_z$$

Relativnost polj

$$E'_y = \gamma(E_y - uB_z)$$
$$B'_z = \gamma(B_z - uE_y/c^2)$$

$$E'_z = \gamma(E_z + uB_y)$$
$$B'_y = \gamma(B_y + uE_z/c^2)$$

$$E'_x = E_x$$

$$B'_x = B_x$$

Elektromagnetni valovi

Osnovne enačbe polja

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0\boldsymbol{j} + \mu_0\varepsilon_0\frac{\partial \boldsymbol{E}}{\partial t}$$

Polje brez nabojev in tokov

$$\boldsymbol{\nabla}^2\boldsymbol{E} - \varepsilon_0\mu_0\frac{\partial^2\boldsymbol{E}}{\partial t^2} = 0$$

$$\boldsymbol{\nabla}^2\boldsymbol{B} - \varepsilon_0\mu_0\frac{\partial^2\boldsymbol{B}}{\partial t^2} = 0$$

$$c^2 = \frac{1}{\varepsilon_0\mu_0}$$

Ravno valovanje

$$\boldsymbol{E} = \boldsymbol{E}_0 \, \mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$
$$\boldsymbol{B} = \boldsymbol{B}_0 \, \mathrm{e}^{\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

$$\boldsymbol{E} = c\boldsymbol{B} \times \boldsymbol{n}$$

Stojno valovanje

$$\boldsymbol{E} = \boldsymbol{E}_0(\boldsymbol{r}) \, \mathrm{e}^{-\mathrm{i}\omega t}$$

$$\boldsymbol{\nabla}^2 \boldsymbol{E}_0 = -k^2 \boldsymbol{E}_0$$
$$k^2 = \omega^2 / c^2$$

Energija valovanja

$$\frac{\partial w}{\partial t} = -\boldsymbol{\nabla} \cdot \boldsymbol{J} - \boldsymbol{j} \cdot \boldsymbol{E}$$
$$w = \frac{\varepsilon_0}{2} \boldsymbol{E}^2 + \frac{1}{2\mu_0} \boldsymbol{B}^2$$
$$\boldsymbol{J} = \frac{1}{\mu_0} \boldsymbol{E} \times \boldsymbol{B}$$

Energija v ravnem valovanju

$$\langle w \rangle = \frac{\varepsilon_0}{2} {E_0}^2 = \frac{1}{2\mu_0} {B_0}^2$$

$$\langle J \rangle = c \langle w \rangle$$

Razklopitev osnovnih enačb

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

$$\boldsymbol{E} = -\boldsymbol{\nabla} U - \frac{\partial \boldsymbol{A}}{\partial t}$$

$$\boldsymbol{\nabla}^2 \boldsymbol{A} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{A}}{\partial t^2} = -\mu_0 \boldsymbol{j}$$

$$\boldsymbol{\nabla}^2 U - \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

Njihova rešitev

$$U_\mathrm{P}(t) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho_\mathrm{Q}(t - r_\mathrm{QP}/c) \, \mathrm{d}V_\mathrm{Q}}{r_\mathrm{QP}}$$

$$\boldsymbol{A}_\mathrm{P}(t) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{j}_\mathrm{Q}(t - r_\mathrm{QP}/c) \, \mathrm{d}V_\mathrm{Q}}{r_\mathrm{QP}}$$

Dipolno sevanje

$$\langle P \rangle = \frac{{p_0}^2 \omega^4}{12\pi\varepsilon_0 c^3}$$

$$\langle I \rangle = \frac{3}{2} \frac{\langle P \rangle}{4\pi} \sin^2 \theta$$

Osnovne enačbe v snovi

$$\nabla \cdot \varepsilon \boldsymbol{E} = \frac{\rho_{\text{free}}}{\varepsilon_0}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \frac{\boldsymbol{B}}{\mu} = \mu_0 \boldsymbol{j}_{\text{free}} + \frac{1}{c^2}\frac{\partial \varepsilon \boldsymbol{E}}{\partial t}$$

Mejni pogoji

$$B_\perp(1) = B_\perp(2)$$
$$\varepsilon_1 E_\perp(1) = \varepsilon_2 E_\perp(2)$$

$$E_\parallel(1) = E_\parallel(2)$$
$$B_\parallel(1)/\mu_1 = B_\parallel(2)/\mu_2$$

Valovanje v dielektriku

$$v = \frac{c}{\sqrt{\varepsilon\mu}}$$

$$n = \sqrt{\varepsilon\mu}$$

Odbojni količnik

$$R_\perp = \left| \frac{n_1 \cos\alpha - n_2 \cos\alpha_2)}{n_1 \cos\alpha + n_2 \cos\alpha_2} \right|^2$$

$$R_\parallel = \left| \frac{n_2 \cos\alpha - n_1 \cos\alpha_2)}{n_2 \cos\alpha + n_1 \cos\alpha_2} \right|^2$$

$$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2$$

Uklon za odprtino

$$u_{\text{P}} \propto \int \exp(\mathrm{i}ks)\,\mathrm{d}S$$

Mrežica tankih rež

$$j(\alpha) = j_0 \left[\frac{\sin({}^1\!/_2 Nka \sin\alpha)}{N \sin({}^1\!/_2 ka \sin\alpha)} \right]^2$$

Široka reža

$$j(\alpha) = j_0 \left[\frac{\sin(ka \sin\alpha)}{ka \sin\alpha} \right]^2$$

Elektroni in ioni

Sila na elektron

$$\boldsymbol{F} = e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Hitrost pospešenega elektrona

$$\frac{1}{2} m v^2 = e U_{\text{A}}$$

Odklon v električnem polju

$$\theta = \frac{eEl}{mv^2}$$

Odklon v magnetnem polju

$$\phi = \frac{eBs}{mv}$$

Brez odklona v prekrižanih poljih

$$v = \frac{E}{B}$$

Relativistični elektroni

$$\theta = \frac{eEl}{\gamma m v^2}$$

$$\phi = \frac{eBs}{\gamma m v}$$

Nepolarna molekula

$$\boldsymbol{p}_\mathrm{e} = \alpha \varepsilon_0 \boldsymbol{E}$$

Plin nepolarnih molekul

$$\frac{M}{\rho}(\varepsilon - 1) = N_\mathrm{A}\alpha$$

Polarna molekula

$$\langle p_\mathrm{e} \rangle = \frac{{p_0}^2}{3kT} E$$

Plin polarnih molekul

$$\frac{M}{\rho}(\varepsilon - 1) = N_\mathrm{A}(\alpha + \frac{{p_0}^2/\varepsilon_0}{3kT})$$

Nemagnetna molekula

$$\boldsymbol{p}_\mathrm{m} = \beta \boldsymbol{B}/\mu_0$$

Plin nemagnetnih molekul

$$\frac{M}{\rho}(\mu - 1) = N_\mathrm{A}\beta$$

Magnetna molekula

$$\langle p_\mathrm{m} \rangle = \frac{{p_0}^2}{3kT} B$$

Plin magnetnih molekul

$$\frac{M}{\rho}(\mu - 1) = N_\mathrm{A}(\beta + \frac{{p_0}^2 \mu_0}{3kT})$$

Elektroni v prevodniku

$$\boldsymbol{v}_\mathrm{drift} = \frac{e\boldsymbol{E}\tau}{m}$$

$$\boldsymbol{j} = ne\boldsymbol{v}_\mathrm{drift} = \sigma \boldsymbol{E}$$
$$\sigma = \frac{ne^2\tau}{m}$$

Prečna napetost

$$U = \frac{1}{ne}\frac{IB}{d}$$

Elektronski oscilatorji

$$n-1=\frac{e^2/m}{2\varepsilon_0({\omega_0}^2-\omega^2)}\frac{N}{V}$$

$$n-1=\frac{N}{V}\sum f_k\frac{e^2/m}{2\varepsilon_0({\omega_k}^2-\omega^2)}$$

Kvantni delci

Fotoelektrični pojav

$$K=h\nu-W,\quad h=6{,}63\cdot10^{-34}\,\mathrm{Js}$$

Energija fotona

$$E=h\nu$$

Gibalna količina fotona

$$G=\frac{E}{c}=\frac{h\nu}{c}=\frac{h}{\lambda}$$

Valovna dolžina elektrona

$$\lambda=\frac{h}{G}=\frac{h}{m\nu}$$

Sipanje alfa na atomskih jedrih

$$b=\frac{1}{2}\left(\frac{Z_1Z_2q^2}{m\nu^2/2}\right)\cot\frac{\theta}{2}$$

$$\frac{\mathrm{d}I/I_0}{\mathrm{d}\Omega}=\frac{1}{16}\frac{N}{V}\left(\frac{Z_1Z_2q^2}{m\nu^2/2}\right)^2\frac{l}{\sin^4\theta/2}$$

Energijski nivoji atomov

$$h\nu=E_2-E_1$$

Kvantizacija krožnic

$$2\pi r_n=n\lambda,\,n=1,2,3\ldots$$

Kvantizacija vrtilne količine

$$L=m\nu r=n\hbar,\,n=1,2,3\ldots$$
$$\hbar=h/2\pi$$

Krožnice v atomu vodika

$$r=\frac{\hbar^2}{mq^2}\cdot n^2=r_\mathrm{B}\cdot n^2,\,n=1,2,3\ldots$$

Obodna hitrost v atomu vodika

$$\alpha=\frac{v}{c}=\frac{q^2}{\hbar c}=1/137$$

Energijski nivoji v atomu vodika

$$E=-\frac{mq^4}{2\hbar^2}\frac{1}{n^2}=E_\mathrm{R}\cdot\frac{1}{n^2}$$

Krožnice ali elipse v atomu vodika

$$E=-\frac{q^2}{2r}=-\frac{q^2}{2a}$$

Velike polosi v atomu vodika

$$a=r_\mathrm{B}\cdot n^2$$

Male polosi v atomu vodika

$$b = \frac{l}{n} a$$

Vrtilne količine elips v atomu vodika

$$L = l\hbar, \quad l = 1, 2, 3 \dots n$$

Slojevita zgradba atomov

$$r_n = r_\mathrm{B} \frac{n^2}{Z - S}$$

$$E_n = E_\mathrm{R} \frac{(Z - S)^2}{n^2}$$

Notranji elektroni

$$\nu = \frac{3}{4} R_\mathrm{y} (Z - 1)^2$$

Orbitalni magnetni moment

$$\mu = \frac{e}{2m} L$$

$$\mu = \frac{e}{2m} \hbar l = \mu_\mathrm{B} l, \quad l = 1, 2, 3 \dots n$$

Njegova precesija

$$\Omega = \frac{e}{2m} B$$

Njegova smerna kvantizacija

$$L_z = m_l \hbar$$
$$\mu_z = m_l \mu_\mathrm{B}$$
$$m_l = -l, -l + 1 \dots -1, 0, 1 \dots l - 1, l$$

Spin in spinski magnetni moment

$$L = s\hbar, \quad s = 1/2$$
$$L_z = m_s \hbar, \quad m_s = -s, +s$$

$$\mu_z = 2 \cdot \frac{e}{2m} L_z = \pm\hbar$$

Celotni magnetni moment

$$L = j\hbar$$
$$\mu = j\mu_\mathrm{B}$$
$$L_z = m_j \hbar$$
$$\mu_z = m_j \mu_\mathrm{B}$$
$$m_j = -j, -j + 1 \dots j - 1, j$$

Valovna mehanika

Valovna funkcija delca

$$\Psi(\boldsymbol{r}, t)$$

Verjetnost lege

$$\frac{\mathrm{d}P}{\mathrm{d}V} = |\Psi|^2$$

$$\int |\Psi|^2 \, \mathrm{d}V = 1$$

Ravni val

$$\Psi(x,t) = A\mathrm{e}^{\mathrm{i}(kx-\omega t)} = A\mathrm{e}^{\mathrm{i}(Gx-Et)/\hbar}$$

Valovni paket

$$\Psi(x) = \frac{1}{\sqrt{(2\pi)}} \int A(k)\mathrm{e}^{\mathrm{i}kx} \, \mathrm{d}k$$

Njegov spekter

$$A(k) = \frac{1}{\sqrt{(2\pi)}} \int \Psi(x)\mathrm{e}^{-\mathrm{i}kx} \, \mathrm{d}x$$

$$\frac{\mathrm{d}P}{\mathrm{d}k} = |A|^2$$

$$\int |A|^2 \, \mathrm{d}k = 1$$

Razmazanost paketa

$$\Delta x^2 = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$
$$\Delta G^2 = \langle (G - \langle G \rangle)^2 \rangle = \langle G^2 \rangle - \langle G \rangle^2,$$

$$\Delta x \, \Delta G \geq \frac{\hbar}{2}$$

Kvantni gibalni zakon

$$\mathrm{i}\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \Psi + W(\boldsymbol{r})\Psi$$

Tok verjetnosti

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$$
$$\boldsymbol{j} = \frac{\hbar}{2m\mathrm{i}} (\Psi^* \boldsymbol{\nabla} \Psi - \Psi \boldsymbol{\nabla} \Psi^*)$$

Stacionarna stanja

$$\Psi(\boldsymbol{r},t) = \psi(\boldsymbol{r}) \, \mathrm{e}^{-\mathrm{i}\omega t}$$

Kvantna amplitudna enačba

$$-\frac{\hbar^2}{2m} \boldsymbol{\nabla}^2 \psi + [W(\boldsymbol{r}) - E]\psi = 0$$

Čisto stanje

$$\Psi(x,t) = \psi_n(x) \, \mathrm{e}^{-\mathrm{i}E_n t/\hbar}$$

Mešano stanje

$$\Psi(x,t) = \sum c_n \psi_n(x) \, \mathrm{e}^{-\mathrm{i}E_n t/\hbar}$$

$$\int {\psi_m}^* \psi_n \, \mathrm{d}V = 1, \text{če } n = m, \text{sicer } 0$$

$$\sum |c_n|^2 = 1$$

$$P(E_n) = |c_n|^2$$

Vpad na stopnico

$$R = \frac{k_1 - k_2}{k_1 + k_2}$$
$$T = \frac{2k_1}{k_1 + k_2}$$

Delec v potencialni jami

$$E_n = \frac{\hbar^2}{2m} \left(\frac{n\pi}{D} \right)^2, \quad n = 1, 2, 3 \ldots$$

$$\psi_n \propto \sin \frac{n\pi}{D} x$$

Harmonični oscilator

$$W = \frac{1}{2} kx^2 = \frac{1}{2} m\omega^2 x^2$$

$$E_n = \hbar\omega \left(n + \frac{1}{2} \right), \quad n = 0, 1, 2, 3 \ldots$$

$$\psi_n \propto H_n \left(\frac{x}{a} \right) \exp \left(- \frac{x^2}{2a^2} \right), \quad a^2 = \frac{\hbar}{m\omega}$$
$$\psi_0 \propto \exp \left(- \frac{x^2}{2a^2} \right)$$

Vodikov atom

$$W = - \frac{q^2}{r}$$

$$E_n = \frac{-mq^4}{2\hbar^2} \frac{1}{n^2}$$
$$\psi_{nlm}(r,\theta,\varphi) \propto R_{nl}(r) P_{lm}(\cos\theta) \Phi_m(\varphi)$$
$$\psi_{100} \propto \exp \left(- \frac{r}{r_B} \right)$$

$$n = 1, 2, 3 \ldots$$
$$l = 0, 1, 2 \ldots n - 1$$
$$m = 0, \pm 1, \pm 2 \ldots \pm l$$

Vrtilna količina

$$-\mathrm{i}\hbar (\boldsymbol{r} \times \boldsymbol{\nabla}) \psi = \boldsymbol{L} \psi$$

$$L^2 = l(l+1)\hbar^2, l = 0, 1, 2, 3, \ldots$$

Projekcija vrtilne količine

$$-\mathrm{i}\hbar \frac{\partial \psi}{\partial \varphi} = L_z \psi$$

$$L_z = m\hbar, m = 0, \pm 1, \pm 2, \ldots \pm l$$

Večelektronski atom

$$\psi(x_1,y_1,z_1,x_2,y_2,z_2) = \psi(1,2)$$

$$\mathrm{d}V = \mathrm{d}x_1\mathrm{d}y_1\mathrm{d}z_1\mathrm{d}x_2\mathrm{d}y_2\mathrm{d}z_2$$

$$\frac{\mathrm{d}P}{\mathrm{d}V} = |\psi|^2$$

$$-[\frac{\hbar^2}{2m}\nabla_1{}^2 + \frac{\hbar^2}{2m}\nabla_2{}^2]\,\psi + W(1,2)\psi = E\psi$$

Neodvisni elektroni

$$W(1,2) = W(1) + W(2)$$
$$\psi(1,2) = u(1)v(2)$$
$$-\frac{\hbar^2}{2m}\nabla_1{}^2 u + W(1)u = E_1 u$$
$$-\frac{\hbar^2}{2m}\nabla_2{}^2 v + W(2)v = E_2 v$$
$$E = E_1 + E_2$$

Molekule, kristali, plini

Rotacija molekule

$$E = \frac{\hbar^2}{2J}\,l(l+1), \quad l = 0, 1, 2, 3\ldots$$

Nihanje dvoatomne molekule

$$E = \hbar\omega_0(n + \frac{1}{2})$$

Porazdelitev molekul po energiji

$$P_i = \frac{1}{Z}\,g_i\,\mathrm{e}^{-E_i/kT}$$
$$Z = \sum g_i\,\mathrm{e}^{-E_i/kT}$$

Nihanje dvoatomnih molekul

$$P_n = \frac{1}{Z}\exp(-\frac{\hbar\omega_0}{kT}\,n)$$

$$Z = \frac{1}{1-\exp(-\hbar\omega_0/kT)}$$

$$\langle E\rangle = \frac{\hbar\omega_0}{\exp(\hbar\omega_0/kT)-1} = \hbar\omega_0\,\langle n\rangle$$

Vrtenje dvoatomnih molekul

$$P_l = \frac{1}{Z}\,(2l+1)\exp-\frac{(\hbar^2/2J)\,l(l+1)}{kT}$$

$$Z \approx \frac{2J}{\hbar^2}\,kT$$

Nihanje atomov v kristalu

$$U = 3NkT\,\frac{\hbar\omega/kT}{\mathrm{e}^{\hbar\omega/kT}-1}$$

$$c_V = 3\,\frac{k}{m_1}\,(\,\frac{\theta}{T}\,)^2\,\frac{\mathrm{e}^{\theta/T}}{(\mathrm{e}^{\theta/T}-1)^2}$$

$$\theta = \frac{\hbar\omega}{k}$$

Fermionska porazdelitev

$$f_i = \frac{N_i}{g_i} = \frac{1}{\mathrm{e}^{(E_i-E_\mathrm{F})/kT}+1}$$

$$\frac{\mathrm{d}n}{\mathrm{d}E} = \frac{g(E)}{\exp{(E-E_\mathrm{F})/kT}+1} = g(E)\,f(E)$$

Elektronski plin v kovini

$$g = \frac{4\pi(2m)^{3/2}}{h^3}\cdot\sqrt{E}$$

$$E_\mathrm{F} = \frac{h^2}{2m}\,(\,\frac{3n}{8\pi}\,)^{2/3}$$

$$W = \frac{3}{5}\,N\,E_\mathrm{F}$$

$$p = \frac{2}{3}\,\frac{W}{V} = \frac{2}{5}\,\frac{NE_\mathrm{F}}{V}$$

$$\frac{1}{2}\,m{v_\mathrm{F}}^2 = E_\mathrm{F}$$

Pogoj za nedegenerirano plazmo

$$\frac{h}{\sqrt{(3mkT)}} \ll \frac{1}{n^{1/3}}$$

Pogoja za nerelativistično plazmo

$$T \ll \frac{mc^2}{k}$$

$$n \ll \frac{\sqrt{8}}{3\pi^2}\,(\,\frac{mc}{\hbar}\,)^3$$

Bozonska porazdelitev

$$f_i = \frac{N_i}{g_i} = \frac{1}{\mathrm{e}^{(E_i-E_\mathrm{F})/kT}-1}$$

$$\frac{\mathrm{d}N}{\mathrm{d}E} = \frac{g(E)}{\exp{(E/kT)}-1} = g(E)\,f(E)$$

Fotoni v votlini

$$g(E) = \frac{8\pi V}{(hc)^3} E^2$$

$$\frac{\mathrm{d}w}{\mathrm{d}E} = \frac{8\pi}{(hc)^3} \frac{E^3}{\mathrm{e}^{E/kT} - 1}$$

$$\frac{\mathrm{d}w}{\mathrm{d}\nu} = \frac{2h\nu^3}{c^2} \frac{1}{\mathrm{e}^{\mathrm{h}\nu/kT} - 1}$$

$$\frac{\mathrm{d}w}{\mathrm{d}\lambda} = \frac{2hc^2}{\lambda^5} \frac{1}{\mathrm{e}^{hc/\lambda kT} - 1}$$

$$p = \frac{1}{3} \frac{W}{V}$$

$$p = a\,T^4$$
$$a = \frac{8\pi^5 k^4}{15c^3h^3}$$

Toplotno sevanje

$$\frac{\mathrm{d}B}{\mathrm{d}\lambda} = \frac{2hc^3}{\lambda^5} \frac{1}{\mathrm{e}^{hc/\lambda kT} - 1}$$

$$\frac{\mathrm{d}j^*}{\mathrm{d}\lambda} = \pi \frac{\mathrm{d}B}{\mathrm{d}\lambda}$$

$$j^* = \sigma T^4$$
$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3}$$
$$a = \frac{4\sigma}{3c}$$

$$\lambda_{\max} = \frac{b}{T}$$
$$b = \frac{hc}{4.97k}$$

Atomska jedra

Centralni trk delca 1 v mirujoč delec 2

$$\frac{v_2'}{v_1} = \frac{2m_1}{m_1 + m_2}$$

Trk delca 1 z jedrom 2

$$\frac{m_2}{m_1} = \frac{\sin\theta_1}{\sin(\theta_1 + \theta_2)}$$

Transmutacija dušika

$${}_2\alpha^4 + {}_7\mathrm{N}^{14} \rightarrow {}_8\mathrm{O}^{17} + {}_1\mathrm{p}^1$$

Odkritje nevtrona

$${}_2\alpha^4 + {}_4\mathrm{Be}^9 \rightarrow {}_6\mathrm{C}^{12} + {}_0\mathrm{n}^1$$

$$m_\mathrm{n} = 1{,}009\,u$$

Vezavna energija jedra

$$E_\mathrm{bind} = [Zm_\mathrm{p} + (A-Z)m_\mathrm{n} - m]c^2$$

Vezavna energija nukleona

$$B = \frac{E_\mathrm{bind}}{A}$$

Razpad alfa

$${}_Z\mathrm{X}^A \rightarrow {}_{Z-2}\mathrm{Y}^{A-4} + {}_2\alpha^4$$

Razpad beta

$${}_Z\mathrm{X}^A \rightarrow {}_{Z+1}\mathrm{Y}^A + \mathrm{e}^- + \nu$$

Razpad gama

$${}_Z\mathrm{X}^A \rightarrow {}_Z\mathrm{X}^A + \gamma$$

Preostala jedra

$$N = N_0\,\mathrm{e}^{-\lambda t}$$

Razpadni čas

$$\tau = \frac{1}{\lambda}$$

Razpolovni čas

$$T_{1/2} = \tau \ln 2$$

Aktivnost vira

$$A = -\frac{\mathrm{d}N}{\mathrm{d}t} = A_0\mathrm{e}^{-\lambda t}$$

Ravnovesje v verigi razpadov

$$\frac{N_\mathrm{A}}{T_\mathrm{A}} = \frac{N_\mathrm{B}}{T_\mathrm{B}} = \frac{N_\mathrm{C}}{T_\mathrm{C}}$$

Absorpcijski radij jedra za nevtrone

$$r = r_0\,\mathrm{A}^{1/3}, \quad r_0 = 1{,}2\,\mathrm{fm}$$

Cepitev težkih jeder

$$\mathrm{n} + \mathrm{U}^{235} \rightarrow \mathrm{X} + \mathrm{Y} + {\sim}2{,}5\,\mathrm{n}$$

Zlivanje lahkih jeder

$$\mathrm{H}^2 + \mathrm{H}^3 \rightarrow \mathrm{He}^4 + \mathrm{n}^1$$

Zvezde in vesolje

Zvezde na glavni veji

$$P \propto {T_\mathrm{E}}^8$$

$$R \propto {T_\mathrm{E}}^2$$

$$P \propto M^4$$

$$T_\mathrm{E} \propto M^{1/2}$$

$$R \propto M$$

$$\langle \rho \rangle \propto \frac{1}{M^2}$$

Masa sredice oblaka

$$M_r = \int_0^r \rho \, 4\pi r^2 \mathrm{d}r$$

Gravitacijska energija oblaka

$$E_\mathrm{G} = -\int \frac{\kappa M_r}{r} \, \mathrm{d}m \sim -\frac{\kappa M^2}{R}$$

Toplotna energija oblaka

$$E_\mathrm{T} = \frac{3}{2} \frac{M}{\bar{m}} kT$$

Sesedna masa oblaka

$$M_\mathrm{J} = \frac{3kT}{2\kappa \bar{m}} R$$

Hidrostatično ravnovesje

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\kappa M_r \rho}{r^2}$$

Tlak v središču

$$p_\mathrm{c} \sim \frac{\kappa M^2}{R^4}$$

Notranja temperatura zvezde

$$T_\mathrm{c} \propto \frac{M}{R}$$

Masni in fotonski tlak

$$\frac{p_\mathrm{rad}}{p_\mathrm{gas}} \propto M^2$$

Vezavna energija zvezde

$$E_\mathrm{bind} = -(E_\mathrm{T} + E_\mathrm{G})$$

Za masni plin

$$E_\mathrm{T} = -\frac{E_\mathrm{G}}{2}$$
$$E_\mathrm{bind} = E_\mathrm{T}$$

Za fotonski plin

$$E_\mathrm{T} = -E_\mathrm{G}$$
$$E_\mathrm{bind} = 0$$

Zlivanje protonov v helij

$$4\mathrm{p} \rightarrow \mathrm{He}^4 + 2\mathrm{e}^+ + 2\nu$$

$$^1\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{H} + \mathrm{e}^+ + \nu + 0.4\,\mathrm{MeV}$$
$$^1\mathrm{H} + {}^2\mathrm{H} \rightarrow {}^3\mathrm{He} + \gamma + 5.5\,\mathrm{MeV}$$
$$^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2({}^1\mathrm{H}) + 12.9\,\mathrm{MeV}$$

Življenjski čas

$$t \propto \frac{1}{M^3}$$

Proizvodnja energije

$$\frac{\mathrm{d}P}{\mathrm{d}r} = \varepsilon\rho 4\pi r^2$$

Difuzijski prenos

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{P}{4\pi r^2 \lambda}$$

Konvektivni prenos

$$\frac{\mathrm{d}T}{\mathrm{d}r} = \frac{\gamma - 1}{\gamma} \frac{T}{p} \frac{\mathrm{d}p}{\mathrm{d}r}$$

Radij bele pritlikavke

$$R \propto \frac{1}{M^{1/3}}$$

Sesedna masa bele pritlikavke

$$M \sim \left(\frac{N}{M}\right)^2 \left(\frac{\bar{h}c}{\kappa}\right)^{3/2}$$

Nihanje svetilnosti kefeid

$$M = -a \lg (P/\mathrm{dan}) - b$$
$$a = 2.4$$
$$b = 1.7$$

Beg galaksij

$$v = H_0 r$$

Trajanje od velikega poka

$$t_0 \sim \frac{1}{H_0}$$

Skalirni faktor vesolja

$$\boldsymbol{r} = a(t)\boldsymbol{R}$$

Skalirna enačba

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi\kappa}{3}\rho - \frac{kc^2}{a^2}$$

$$H = \frac{a'}{a}$$

Gostotna enačba

$$\rho' + 3\frac{a'}{a}\left(\rho + \frac{p}{c^2}\right) = 0$$

Raztegovanje svetlobe

$$\lambda \propto a$$

Ravno masno dominirano vesolje

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\rho(t) = \frac{\rho_0}{a^3}$$

Ravno sevalno dominirano vesolje

$$a(t) = \left(\frac{t}{t_0}\right)^{1/2}$$

$$\rho(t) = \frac{\rho_0}{a^4}$$

Kritična gostota vesolja

$$\rho_c = \frac{3H^2}{8\pi\kappa}$$

Ohlajanje prasevanja

$$T \propto \frac{1}{a}$$